## NUMERICAL ESTIMATION OF THE EFFICIENCY OF THERMAL INSULATION FOR THE MAIN ELEMENTS OF AN AEROSPACE VEHICLE

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We present results of a numerical analysis of the fields of temperature and thermal stresses in a multilayer wall of the fuel tank of an aerospace vehicle.

It has been established [1] that one can obtain adequate weight characteristics for an aerospace vehicle (ASV) only in the case of maximum use of composite materials in both load-bearing and heat-insulating and heat-shielding elements of its structure.

Among the basic problems to be solved in connection with the introduction of composite materials into the structure of an ASV is the problem [1] of ensuring the efficiency of a multilayer wall of the fuel tank that is the load-bearing structure of the ASV fuselage and operates in the presence of large temperature drops over its thickness. The efficiency of this wall is largely determined by the reliability of a layer of low-temperature thermal insulation, for the manufacture of which foam plastic offers the greatest promise [1]. The mechanism of the operation of foam plastic as a thermal-insulation material depends in many respects on its structure. An important factor for ensuring the needed thermal-insulation characteristics is the absence of noncommunicating pores or cracks. Thus, for example, the formation of cracks in foam plastic leads to a sharp change in the mechanism of the fuel tank mass transfer and deterioration of the properties of both this material and the entire multilayer wall of the fuel tank of an ASV [1].

The objective of the present work was a numerical analysis of the level of thermal stresses appearing in a multilayer wall of the fuel tank of an ASV under typical conditions of operation of the vehicle and, on the basis of this analysis, estimate of the efficiency of the layer of low-temperature insulation and the structure of the fuel tank of the ASV as a whole.

We consider a multilayer wall of the fuel tank of an ASV that includes the following basic elements: a force layer, a sealing interlayer, low-temperature thermal insulation, and high-temperature thermal insulation.

The force layer is in contacts with a liquid fuel that is held at cryogenic temperatures (20-30 K). The surface of the high-temperature heat shielding coating is exposed to air flow. The conditions of heat exchange on this boundary depend strongly on the mode of operation of the ASV (refueling, ascent, descent, landing, discharge of the remaining fuel, etc.).

For the numerical analysis we selected typical materials used for the manufacture of each of the layers [1, 2]. The force layer is made of steel, polyethylene is used for the sealing interlayer, the low-temperature thermal insulation is made of foam plastic, and the high-temperature thermal insulation is made of carbon-filled plastic. The thermophysical and physicomechanical properties of these materials are given in [2-5].

We consider the problem of the thermal and stress-strain state of a hollow, multilayer cylinder whose internal channel is filled with a low-temperature liquid. The process of heat propagation in the cylinder is calculated using a model of unsteady-state heat conduction. To estimate the level of the thermal stresses that appear, we use a model of a quasistatic approximation of a plane stress-strain state of an infinitely long cylinder. Profiles of the stresses that appear are calculated on the basis of the obtained temperature profiles for each moment of time.

The system of equations, conditions of conjugation on the boundaries of the layers, and boundary and initial conditions that describes the investigated process within the framework of the assumptions made has the form [6, 7]

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$$\rho_i c_i \left( T \right) \frac{\partial T}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda_i \left( T \right) r \frac{\partial T}{\partial r} \right) , \tag{1}$$

$$\sigma_{z} = \frac{E_{i}}{(1+\mu_{i})(1-2\mu_{i})} \left[ (1-\mu_{i}) \varepsilon_{z} + \mu_{i} \varepsilon_{r} + \mu_{i} \varepsilon_{q} - (1+\mu_{i}) \alpha_{i} t \right],$$
(2)

$$\sigma_r = \frac{E_i}{(1+\mu_i)(1-2\mu_i)} \left[ (1-\mu_i) \varepsilon_r + \mu_i \varepsilon_q + \mu_i \varepsilon_z - (1+\mu_i) \alpha_i t \right],$$
(3)

$$\sigma_q = \frac{E_i}{(1+\mu_i)(1-2\mu_i)} \left[ (1-\mu_i) \,\epsilon_q + \mu_i \epsilon_z + \mu_i \epsilon_r - (1+\mu_i) \,\alpha_i t \right], \tag{4}$$

$$\varepsilon_z = \text{const}$$
, (5)

not known a priori

$$\varepsilon_r = \frac{du_r}{dr} \,, \tag{6}$$

$$\varepsilon_q = \frac{u_r}{r},\tag{7}$$

$$\frac{d}{dr}\left[\frac{1}{r} \frac{d\left(u_{r}r\right)}{dr}\right] = \frac{1+\mu_{i}}{1-\mu_{i}}\alpha_{i}\frac{dt}{dr},$$
(8)

at  $\tau = 0$ 

$$T = T_0, (9)$$

at  $r = R_{in}$ 

$$\lambda (T) \frac{\partial T}{\partial r} = h_{\rm in} (T) (T_{\rm in} - T) , \qquad (10)$$

$$\sigma_r = -p_0, \qquad (11)$$

at  $r = R_{ex}$ 

$$\lambda (T) \frac{\partial T}{\partial r} = h_{\rm ex} (T) (T_{\rm ex} - T) , \qquad (12)$$

$$\sigma_r = -p_0 \,, \tag{13}$$

at  $r = r_{2,i} = r_{1,i+1}$ 

$$\lambda_{i}(T) \left. \frac{\partial T}{\partial r} \right|_{r=0} = \lambda_{i+1}(T) \left. \frac{\partial T}{\partial r} \right|_{r=0},$$
(14)

$$\sigma_{r,i} = \sigma_{r,i+1} , \tag{13}$$

$$u_{r,i} = u_{r,i+1}$$
, (16)

$$\varepsilon_{z,i} = \varepsilon_{z,i=1} \,. \tag{17}$$

Equalities (5) and (17) yield constancy of  $\varepsilon_z$  throughout the entire computational region.

Closure of the system of equations is achieved by stipulating absence of a longitudinal force in the transverse section of the cylinder, which in the case of a cylinder of finite length is satisfied at a sufficient distance from the cylinder end faces:

$$\int_{R_{\rm in}}^{R_{\rm ex}} \sigma_z r dr = 0.$$
<sup>(18)</sup>

The nonstationary differential equation of heat conduction is approximated by an implicit four-point difference scheme on a grid that is uniform for each layer. At points on the boundaries of the layers the values of the thermal conductivity coefficient and the specific heat are calculated as the arithmetic mean of heir values in the boundary layers [8]. This technique makes it possible, without disturbing the conservateness of the scheme, to organize a continuous computation over space without isolating boundary points. The obtained difference analog of the original differential equation is solved by the sweeping method with the use of iterations [8].

To find the values of  $\sigma_r$ ,  $\sigma_z$ , and  $\sigma_q$ , we use the following method.

Double integration of Eq. (8) from the minimum radius of the layer  $r_{1,i}$  to the running radius r yields

$$u_r = \frac{1}{r} \frac{1 + \mu_i}{1 - \mu_i} \int_{r_{1,i}}^r \alpha_i \, tr dr + r C_{1,i} + \frac{C_{2,i}}{r} \,. \tag{19}$$

Substituting the resulting expression into Eqs. (6), (7) and the latter into Eqs. (2)-(4), we obtain the values of the stresses expressed in terms of the integration constants  $C_{1,i}$ ,  $C_{2,i}$  (in the general case, different for each layer):

$$\sigma_r = \frac{E_i}{(1+\mu_i)(1-2\mu_i)} \left[ -\frac{(1-2\mu_i)(1+\mu_i)}{1-\mu_i} \frac{1}{r^2} \int_{r_{1,i}}^r \alpha_i tr dr + C_{1,i} - (1-2\mu_i) \frac{C_{2,i}}{r^2} + \mu_i \varepsilon_z \right], \quad (20)$$

$$\sigma_{z} = \frac{E_{i}}{(1+\mu_{i})(1-2\mu_{i})} \left[ (1-\mu_{i}) \epsilon_{z} - \frac{(1-2\mu_{i})(1+\mu_{i})}{1-\mu_{i}} \alpha_{i}t + 2\mu_{i}C_{1,i} \right],$$
(21)

$$\sigma_{q} = \frac{E_{i}}{(1+\mu_{i})(1-2\mu_{i})} \left[ \frac{(1-2\mu_{i})(1+\mu_{i})}{1-\mu_{i}} \frac{1}{r^{2}} \int_{r_{1,i}}^{r_{2,i}} \alpha_{i} tr dr + C_{1,i} + (1-2\mu_{i}) \frac{C_{2,i}}{r^{2}} - \frac{(1-2\mu_{i})(1+\mu_{i})}{1-\mu_{i}} \alpha_{i}t + \mu_{i}\epsilon_{z} \right].$$
(22)

Boundary conditions (11), (13), the conditions of conjugation on the boundaries of the layers (15), (16), and integral (18) are used as a system of linear equations for the integration constants  $C_{1,i}$ ,  $C_{2,i}$  and the unknown relative deformation  $\varepsilon_z$ . The system of linear equations obtained is written in matrix form and is solved according to Kramer's rule in terms of the corresponding determinants [9].



Fig. 1. Distribution of temperature (a) tangential (b), axial (c), and radial (d) stresses over the radius at different times (T, K; r, m;  $\sigma_q$ ,  $\sigma_z$ ,  $\sigma_r$ , MPa): 1)  $\tau = 230 \text{ sec; } 2) 1030; 3) 2310; 4)$  tensile strength limit of foam plastic.

To implement the described method of solving the initial system of differential equations a computing program was developed in the algorithmic language FORTRAN for an IBM computer.

In conducting calculations, we used a real technological scheme for filling the fuel tank of the ASV. According to this scheme, at the time  $\tau = 0$  the first portion of the liquid fuel at a cryogenic temperature T = 20 K enters the tank and comes into contact with the material of the force layer. As is seen from the above system of equations and boundary conditions, the problem is considered under a number of assumptions. The main ones are the assumptions that the layer of low-temperature thermal insulation is a homogeneous material with intercommunicating pores and without any damage of the crack type and that at the stage of filling of the fuel tank of the ASV no physicochemical conversions (condensation of air, formation of ice, etc.) occur in the low-temperature thermal-insulation layer.

We carried out numerical investigations for the following values of the main parameters that characterize the structure considered: the thicknesses of the layers beginning from the inner radius 0.2, 1.5, 230, and 2 mm, the heat transfer coefficient on the inner and outer boundaries 5 W/( $m^2 \cdot K$ ), the temperature of the liquid in the tank 20 K, the temperature of the air on the outer boundary of the structure 293 K.

Results of calculations in the form of the distribution of temperature and stresses over the coordinate r at different times are given in Fig. 1. Attention is mainly paid to analysis of the operation of foam plastic, which plays the role of low-temperature thermal insulation, and therefore results are presented for the layer of low-temperature thermal insulation. The thermal stresses in the remaining layers of the structure do not attain values that would be limiting for the given materials.

It should be noted that the results of calculations given in the present work were obtained for the regime of filling the ASV. The takeoff, flight, and landing of such a vehicle take place under conditions where the temperature of the surface of the layer of high-temperature heat shielding (carbon-filled plastic or a carbon-carbon material) attains 1300 K [10]. An estimate of the efficiency of this layer under conditions of high temperatures can be made with the use of well-known methods [11].

Temperature distributions over the radius for different times are presented in Fig. 1a.

Figure 1b displays distributions of tangential stresses over r for the same times. If is seen that as the temperature decreases, the value of  $\sigma_q$  at the interface between the foam plastic and the sealant grows. It is seen from the figure that values  $\sigma_q > \sigma_{\text{lim}}$  are reached already at a temperature on the foam plastic-sealant interface of  $\cong 240$  K, and formation of cracks in the foam plastic oriented in the r - z plane is possible. A further decrease in the temperature leads to an increase in  $\sigma_q$  and in the difference  $\sigma_q - \sigma_{\text{lim}}$ . Distributions of the values of axial stresses over the radius are given in Fig. 1c. It is seen that on the foam plastic-sealant interface the values of  $\sigma_z$  also exceed  $\sigma_{\text{lim}}$  already at a temperature of  $\cong 240$  K and, correspondingly, in the case of actual filling of the fuel tank of the ASV at such temperatures cracking of foam plastic with the formation of cracks oriented in the r - q plane is possible.

Comparing the values of  $\sigma_q$  and  $\sigma_z$  and taking into account that, according to [12], axial stresses in long cylinders are generally equal to tangential ones, we can conclude that there is a high probability of formation of a network of cracks in the foam plastic layer. Such cracks not only worsen the physicomechanical characteristics of the structure, but, and this is the major point, they drastically, as follows from [1, 2], alter the mechanism of heat and mass transfer in the layer of low-temperature thermal insulation. In the presence of cracks in the layer of low-temperature heat insulation, heat transfer in this layer occurs not only by heat conduction, as in the case of intercommunicating pores, but also due to filtration of the air in foam plastic in the direction of the zone of lowtemperatures near the force layer and its condensation. In the presence of the sealant between the layer of lowtemperature heat insulation and high-temperature heat shield, the aforesaid, in turn, leads to a considerable drop in the pressure in the foam plastic, and this, under certain conditions, may damage the sealant and the external heat-shielding layer. Thus, the numerical analysis carried out makes it possible to conclude that in estimating the efficiency of such a high-temperature apparatus as an ASV, it is necessary to carry out simultaneous calculations of both the thermal and stress-strain state of the structure with account for the actual mechanisms underlying the operation of each structural element.

Distributions of real stresses over the radius are presented in Fig. 1d. It is seen from the figure that the values of  $\sigma_r$  in any cross section over r are much smaller than the compressive and tensile strength limits of the foam plastic. These results allow the conclusion that the radial stresses appearing in the foam plastic will not lead to cracking of the material.

On the basis of the reaults obtained we can make the following conclusions:

1. The conditions of operation of the low-temperature thermal insulation of the fuel tank of an ASV are such that already in cooling the structure below 240 K axial and tangential thermal stresses appear on the foam plastic-sealant interface that exceed in magnitude the tensile strength limit of the foam plastic. This should lead to the appearance of cracks in this material in the r - z and r - q planes.

2. A further decrease in temperature to  $T \approx 30$  K on the foam plastic-sealant interface entails formation and development of cracks in the foam plastic layer to a large depth.

3. Formation of cracks in the layer of low-temperature thermal insulation of the fuel tank leads to a change in the mechanism of heat and mass transfer in this layer, and this must be taken into account in analyzing the operation of the entire structure, because filtration of air in the foam plastic during both "cooling" of the structure [1, 2] and its "warming up" leads to a more intense change in the internal pressure in the foam plastic compared to the version with no cracks or intercommunicating pores.

## NOTATION

*i*, number of the layer; 0, initial value; in, internal radius of the cylinder; ex, external radius of the cylinder; *r*, *z*, *q*, cylindrical coordinates;  $\sigma_r$ ,  $\sigma_z$ ,  $\sigma_q$ , radial, axial, and tangential stresses; *T*, temperature;  $t = T - T_0$ , difference between the current and initial temperatures;  $\tau$ , time; *c*, specific heat;  $\lambda$ , thermal-conductivity coefficient; *h*, heat transfer coefficient; *E*, Young's modulus;  $\mu$ , Poisson coefficient;  $\alpha$ , temperature coefficient of linear expansion;  $u_r$ , radial deformation;  $\varepsilon$ , relative deformation;  $C_1$ ,  $C_2$ , integration constants;  $r_{1,i}$ ,  $r_{2,i}$ , minimum and maximum radii of the *i*-th layer.

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